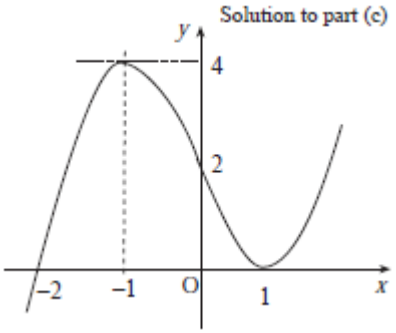
00 P2 A1	**5C, 5C** •1 $y' = \ldots$ •2 $3x^2 - 6x + 2$ •3 $y'(1) = -1$ •4 $y(1) = 0$ •5 $y - 0 = -1(x - 1)$ •6 $2x - 4 = x^3 - 3x^2 + 2x$ •7 $x^3 - 3x^2 + 4 = 0$ •8 … 1 –3 0 4 / … … … / … … … … •9 Identify $x = -1$ from working •10 $(-1, -6)$
01 P2 Q1	**3C, 2C** •1 $-2 \mid 2 \quad 1 \quad k \quad 2$ / $-4 \quad \ldots \quad \ldots$ / $2 \quad -3 \quad \ldots \quad \ldots$ •2 $-2 \mid 2 \quad 1 \quad k \quad 2$ / $-4 \quad 6 \quad -2(k+6)$ / $2 \quad -3 \quad k+6 \quad -2(k+6)+2$ •3 $k = -5$ **OR** •1 $f(-2) = 2(-2)^3 + \ldots\ldots$ •2 $2(-2)^3 + (-2)^2 - 2k + 2$ •3 $k = -5$ •4 $2x^2 - 3x + 1$ *or* $2x^2 + 3x - 2$ *or* $x^2 + x - 2$ •5 $(2x-1)(x-1)$ *or* $(2x-1)(x+2)$ *or* $(x+2)(x-1)$ *and* $x = -2, \frac{1}{2}, 1$
02 P2 Q3	**5C, 3C, 2B**

	•1 $f'(x) = \ldots\ldots$ •2 $6x^2 - 14x + 4$ •3 $6x^2 - 14x + 4 = 0$ •4 $(3x-1)(x-2)$ •5 $x = \frac{1}{3}$ •6 … 2 −7 4 4 / … … … … / … … … 0 •7 $2x^2 - 3x - 2$ •8 $(x-2)(2x+1)(x-2)$ •9 $A(-\frac{1}{2}, 0)$ •10 $x < -\frac{1}{2}$
3.(JAN) 02 P1	**5C** •1 use $f(2)$ or $f(-3)$ or start appr.synth. division •2 $2c + d + 32$ •3 $-3c + d - 63$ •4 $2c + d = -32$ **and** $-3c + d = 63$ or equiv •5 $(c, d) = (-19, 6)$
4.(JAN) 02 P2	**3C** •1 evaluating $f(b)$ for any b making $f(b) + ve$ or $-ve$, $0 \le b \le 0.5$ •2 evaluating $f(c)$ for any c making $f(c) - ve$ or $+ve$, $0 \le c \le 0.5$ **and** evaluating $f(d)$ for any d lying between b and c. •3 $a = 0.2$ x: 0, 0.1, 0.2, 0.25, 0.3, 0.4, 0.5, 1 f(x): 1, 0.625, 0.216, –0.03125, –0.296, –0.872, –1.5, –5
03 P2 Q1	**4C** •1 2 6 −5 −17 6 / 12 / 6 •2 2 6 −5 −17 6 / 12 14 −6 / 6 7 −3 0 •3 $6x^2 + 7x - 3$ •4 $(x-2)(2x+3)(3x-1)$ **stated explicitly**
04 P1 Q2	**5C, 1C**

	•[1] *know to find* $f(-1)$ •[2] -1 | $1 \quad -1 \quad -5 \quad -3$ / $\quad\quad -1$ / $\quad 1$ •[3] -1 | $1 \quad -1 \quad -5 \quad -3$ / $\quad\quad -1 \quad 2 \quad 3$ / $\quad 1 \quad -2 \quad -3 \quad 0$ •[4] $x^2 - 2x - 3$ •[5] $(x+1)(x+1)(x-3)$ •[6] $(-1,0)$ **OR** •[1] *know to find* $f(-1)$ •[2] $f(-1) = (-1)^3 - (-1)^2 - 5(-1) - 3 = 0$ •[3] *a strategy for finding the quadratic factor* *eg inspection, long division, synthetic division*
05 P1 Q8	**5C, 2C, 5B** •[1] *eg* 3 | $2 \quad -7 \quad 0 \quad 9$ •[2] *eg* 3 | $2 \quad -7 \quad 0 \quad 9$ / $\quad\quad 6 \quad -3 \quad -9$ / $2 \quad -1 \quad -3 \quad 0$ •[3] remainder is zero so $(x-3)$ is a factor •[4] $2x^2 - x - 3$ •[5] $(x-3)(2x-3)(x+1)$ **stated explicitly** **OR** •[1] $f(3) = \ldots$ •[2] $f(3) = 2\times 3^3 - 7\times 3^2 + 9 = 54 - 63 + 9 = 0$ •[3] *eg* 3 | $2 \quad -7 \quad 0 \quad 9$ / $\quad\quad 6$ / $2 \quad -1 \quad -3 \quad 0$ •[4] $2x^2 - x - 3$ •[5] $(x-3)(2x-3)(x+1)$ •[6] $(0,9)$ •[7] $(-1,0),\left(\frac{3}{2},0\right),(3,0)$ •[8] $6x^2 - 14x = 0$ •[9] $x = 0$ *or* $x = \frac{14}{6}$ •[10] $f(-2) = -35$ ***OR*** $f(2) = -3$ •[11] *greatest value* $= 9$ •[12] *least value* $= -35$ **OR** •[3] $(x-3)(2x^2 \ldots\ldots\ldots)$ •[4] $(x-3)\left(2x^2 - x - 3\right)$ •[5] $(x-3)(2x-3)(x+1)$ **OR** •[10] *nature table showing* $x = 0$ *is max. tp and the greatest (maximum) value is* 9 •[11] $f(-2) = -35$ ***OR*** $f(2) = -3$ •[12] *least value* $= -35$
05 P2 Q11	**1C, 7A**

	$\bullet^1$ $f(-1) = -1 + p - p + 1 = 0$ $\bullet^2$ -1 | 1 p p 1 / -1 $1-p$ -1 / 1 $p-1$ 1 0 $\bullet^3$ $x^2 + (p-1)x + 1 = 0$ $\bullet^4$ "$b^2 - 4ac$" $\bullet\bullet\bullet$ "≥ 0" $\bullet^5$ $(p-1)^2 - 4$ $\bullet^6$ $(p-3)(p+1)$ $\bullet^7$ $p = 3,\ p = -1$ $\bullet^8$ $p \leq -1,\ p \geq 3$
08 P1 Q21	**6C, 5C, 4C/B** $\bullet^1$ $f'(x) = 0$ $\bullet^2$ $3x^2 - 3$ $\bullet^3$ x | $\bullet^3$ -1 | $\bullet^4$ 1 $\bullet^4$ y | 4 | 0 $\bullet^5$ f' | $\ldots$ $\bullet^5$ -1 $\ldots$ | $\ldots$ $\bullet^6$ 1 $\ldots$ $\bullet^5$ | $+$ 0 $-$ | $-$ 0 $+$ $\bullet^6$ | max at $x =$ -1 | min at $x =$ 1 $\bullet^7$ know to use $x = 1$ $\bullet^8$ $1 - 3 + 2 = 0 \Rightarrow x - 1$ is a factor $\bullet^9$ $(x-1)(x^2 \ldots)$ $\bullet^{10}$ $(x-1)(x^2 + x - 2)$ $\bullet^{11}$ $(x-1)(x-1)(x+2)$ *stated explicitly* $\bullet^{12}$ $(0, 2)$ $\bullet^{13}$ $(-2, 0),\ (1, 0)$ $\bullet^{14}$ Sketch with turning pts marked $\bullet^{15}$ Sketch with $(0, 2)$ or $(-2, 0)$ Solution to part (c)  **Alternative Method:** $\bullet^7$ to $\bullet^{10}$ $\bullet^7$ 1 | 1 0 −3 2 / —— $\bullet^8$ 1 | 1 0 −3 2 / 1 1 −2 / 1 1 −2 0 $\bullet^9$ $f(1) = 0$ *so* $(x-1)$ *is a factor* $\bullet^{10}$ $x^2 + x - 2$

10 P1 Q22

know to use $x=1$ $\bullet^1$

1	2	1	−8	5	

complete evaluation $\bullet^2$

1	2	1	−8	5
		2	3	−5
	2	3	−5	0

state conclusion $\bullet^3$ $(x-1)$ is a factor

find quadratic factor $\bullet^4$ $2x^2+3x-5$

factorise completely $\bullet^5$ $(x-1)(x-1)(2x+5)$

OR

$\bullet^1$ know to use $x=1$

$\bullet^2$ $2+1-8+5=0$

$\bullet^3$ $(x-1)$ is a factor

$\bullet^4$ $(x-1)(2x^2+3x-5)$

$\bullet^5$ $(x-1)(x-1)(2x+5)$

Note

Unacceptable statements : $x=1$ is a factor, $(x+1)$ is a factor, $x=1$ is a root, $(x-1)$ is a root etc.

state solutions $\bullet^6$ $x=1$ **and** $x=-\frac{5}{2}$ or $-2\cdot5$ or $-2\frac{1}{2}$

set $y_{CURVE} = y_{LINE}$ $\bullet^7$ $2x^3+x^2-6x+2=2x-3$ *stated explicitly*

express in standard form $\bullet^8$ $2x^3+x^2-8x+5$

compare with (a) or factorise $\bullet^9$ $(x-1)(x-1)(2x+5)$ $\Big\}=0$

identify x_G $\bullet^{10}$ $x=1$

evaluate y_G $\bullet^{11}$ $y=-1$

OR

know to and differentiate curve $\bullet^7$ $6x^2+2x-6$

set derivative to gradient of line $\bullet^8$ $6x^2+2x-6=2$

solve quadratic equation $\bullet^9$ $x=-\frac{4}{3}$ and 1

process to identify x_G $\bullet^{10}$ at $x=1$ evaluate y_{CURVE} **and** y_{LINE}

complete to $y_{CURVE} = y_{LINE}$ $\bullet^{11}$ $y=-1$ from both curve and line

state solution $\bullet^{12}$ $\left(-\frac{5}{2},-8\right)$

11 P2 Q2

interpret notation $\bullet^1$ $g(x^3-1)$

complete process $\bullet^2$ $3(x^3-1)+1$

substitute and complete $\bullet^3$ $3(x^3-1)+1+x(4x-5)$

$= 3x^3+4x^2-5x-2$ **stated explicitly**

know to use $x=1$ $\bullet^4$

1	3	4	−5	−2

complete evaluation $\bullet^5$

1	3	4	−5	−2
		3	7	2
	3	7	2	0

	state conclusion	$\bullet^6$ "remainder is zero so $(x-1)$ is a factor"
	OR	$\bullet^4$ know to use $x=1$ $\bullet^5$ $3+4-5-2=0$ $\bullet^6$ $(x-1)$ is a factor
	find quadratic factor	$\bullet^7$ $3x^2+7x+2$ **stated, or implied**
	factorise completely	$\bullet^8$ $(x-1)(3x+1)(x+2)$ **stated explicitly**
	solve equation in (d)	$\bullet^9$ -2, $-\frac{1}{3}$ and 1
13 P2 Q3	know to use $x = 1$ in synthetic division	$\bullet^1$ 1 | 1 3 1 -5 1 4 5
	complete evaluation	$\bullet^2$ 1 4 5 0
	state quadratic factor	$\bullet^3$ $x^2 + 4x + 5$
	valid reason for irreducible quadratic	$\bullet^4$ $(x-1)(x^2+4x+5)$ with valid reason
	start to differentiate	$\bullet^5$ two non-zero terms correct
	complete derivative and equate to 0	$\bullet^6$ $4x^3 + 12x^2 + 4x - 20 = 0$
	factorise	$\bullet^7$ $4(x-1)(x^2+4x+5)$
	process for x	$\bullet^8$ $x = 1$
	justify nature and state conclusion	$\bullet^9$ nature table **and** minimum

x	..... 1 .....
$\frac{dy}{dx}$	$-$ 0 $+$
	Min

14 P1 Q22	know to use $x=-1$ and obtain an equation	$\bullet^1$ $6(-1)^3+7(-1)^2+a(-1)+b=0$
	know to use $x=2$ and obtain an equation	$\bullet^2$ $6(2)^3+7(2)^2+a(2)+b=72$
	process equations to find one value	$\bullet^3$ $a=-1$ or $b=-2$
	find the other value	$\bullet^4$ $b=-2$ or $a=-1$
		Alternative Method for $\bullet^1$ and $\bullet^2$
		$\bullet^1$ $-1 \mid 6 \quad 7 \quad a \quad b$ $\quad\quad -6 \quad -1 \quad -a+1$ $\quad 6 \quad 1 \quad a-1 \quad b-a+1=0$
		$\bullet^2$ $2 \mid 6 \quad 7 \quad a \quad b$ $\quad\quad 12 \quad 38 \quad 2a+76$ $\quad 6 \quad 19 \quad a+38 \quad 2a+b+76=72$
	substitute for a and b and know to divide by $x+1$	$\bullet^5$ $(6x^3+7x^2-x-2)\div(x+1)$ Stated or implied by $\bullet^6$
	obtain quadratic factor	$\bullet^6$ $(x+1)(6x^2+x-2)$
	complete factorisation	$\bullet^7$ $(x+1)(3x+2)(2x-1)$

ANSWERS **Pre 2000 – Factorising Polynomials**

1	$\bullet^1$ *strat*: make 2 trial divisions or 2 trial evaluations $\bullet^2$ first linear factor $\bullet^3$ quadratic factor $\bullet^4$ other linear factors $(x-1)(2x+1)(x+3)$	
2	$\bullet^1$ $f(2)=16+4-26+6=0$ ***or*** the appearance of a '0' at the end of the 3rd line in the table below	$\bullet^2$ $2 \mid 2 \quad 1 \quad -13 \quad 6$ $\quad\quad 4 \quad 10 \quad -6$ $\quad 2 \quad 5 \quad -3 \quad 0$ $\bullet^3$ $2x^2+5x-3$ $\bullet^4$ $-3, \frac{1}{2}$
3	$\bullet^1$ evaluating $f(k)$ for any integer by any method $\bullet^2$ find 1 value of k s.t. $f(k)=0$ *e.g.* $f(1)$ *or* $f(-2)$ *or* $f(5)$	$\bullet^3$ quad factor *e.g.* $x^2-3x-10$ $\bullet^4$ $(x-1)(x+2)(x-5)$
4	$\bullet^1$ $x(x^3-1)$ $\bullet^2$ synthetic division or eval. $f(k)$ $\bullet^3$ linear factor $=(x-1)$ $\bullet^4$ $x(x-1)(x^2+x+1)$	***OR*** $\bullet^1$ synthetic division or eval. $f(k)$ $\bullet^2$ linear factor $=(x-1)$ $\bullet^3$ cubic factor $=(x^3+x^2+x)$ $\bullet^4$ $x(x-1)(x^2+x+1)$

5	$\bullet^1$ $f(3)=2\times3^3+3\times3^2-23\times3-12$ | or equivalent division $\bullet^2$ $=0$ $\bullet^3$ $2x^2+9x+4$ $\bullet^4$ $(x-3)(2x+1)(x+4)$
6	$\bullet^1$ looking for $f(x)=\ldots\ldots=0$ $\bullet^2$ $x=2$ **explicitly stated** $\bullet^3$ $2x^2+x+4$ $\bullet^4$ $b^2-4ac=1-4\times2\times4$ $\bullet^5$ $b^2-4ac<0$ means no real roots
7	(*a*) $\bullet^1$ $f(g(x))=f(x-1)$ $\bullet^2$ $(x-1)^3-2(x-1)^2-5(x-1)+6$ $\bullet^3$ $(x-1)^3=x^3-3x^2+3x-1$ $\bullet^4$ $-2x^2+4x-2-5x+5+6$ and completing argument (*b*) $\bullet^5$ *first "0" e.g.* $2 \mid 1\ -5\ \ 2\ \ 8$ / $2\ -6\ -8$ / $1\ -3\ -4\ \ 0$ $\bullet^6$ $x^2-3x-4=(x+1)(x-4)$ $\bullet^7$ $(x-2)(x+1)(x-4)$ (*c*) $\bullet^8$ denominator $(=(x-2)(x+1)(x-4))\neq0$ $\bullet^9$ $-1,\ 2,\ 4$

ANSWERS **Pre 2000 – Finding Coefficients**

1	$\bullet^1$ $f(-3)=-54-27-3p+30$ **or** synth. division e.g. $-3 \mid 2\ \ -3\ \ p\ \ 30$ / $-6\ \ 27\ \ -3p-81$ / $2\ \ -9\ \ p+27\ \ -3p-51$ $\bullet^2$ $p=-17$ **and** $-3p-51=0$ $\bullet^3$ $2x^2-9x+10$ $\bullet^4$ $2,\frac{5}{2}$
2	$\bullet^1$ $f(2)=8+4k-8-12$ $\bullet^2$ $f(2)=0$ $\bullet^3$ $k=3$ $\bullet^1$ correct use of division $\bullet^2$ remainder $=4k-12$ $\bullet^3$ $k=3$
3	$\bullet^1$ $f(2)=114$ $\bullet^2$ $f(-1)=0$ $\bullet^3$ $4p+2q=78$ $\bullet^4$ $p-q=-15$ $\bullet^5$ $p=8, q=23$
4	$\bullet^1$ *strat*: e.g. find $f(-3)$ $\bullet^2$ $f(-3)=0$ $\bullet^3$ $p=-7$

ANSWERS **Pre 2000 Questions – Solving Polynomial Equations**

1

(a) $\bullet^1$ strategy

eg 2 | 2 | 1 | −13 | *a*

	4	10	−6
2	5	−3	0

or $f(2) = 0 = 16 + 4 - 26 + a$

$\bullet^2$ $a = 6$

$\bullet^3$ $(0, 6)$

(b) $\bullet^4$ $2x^2 + 5x - 3$

$\bullet^5$ $(x+3)(2x-1)$

$\bullet^6$ $x = -3,\ \frac{1}{2}$

$\bullet^7$ $(-3, 0),\ \left(\frac{1}{2}, 0\right)$

2

(a) $\bullet^1$ equating expressions for y

$\bullet^2$ re-arranging cubic..... "..." = 0

$\bullet^3$ strategy for solving cubic

$\bullet^4$ first linear factor

$\bullet^5$ quadratic factor

$\bullet^6$ $x = -2,\ 2$

$\bullet^7$ intersection at $(-2, 5)$

(b) $\bullet^8$ double root $\Rightarrow$ tangency **or** $y'(-2) = -4 =$ gradient of line

3

(a) $\bullet^1$ suitable choice of scales

$\bullet^2$ sketch of $y = x^3$ from $x = -3$ to $x = 3$

$\bullet^3$ sketch of $y = 6x + 1$ from $x = -3$ to $x = 3$

(b) $\bullet^4$ 3 roots

(c) $\bullet^5$ 1st estimate: between 2 and 3

$\bullet^6$ 2nd estimate: between 2.5 and 2.6

$\bullet^7$ 3rd estimate: between 2.53 and 2.534

$\bullet^8$ 2.53

ANSWERS **Pre 2000 Questions - Polynomials and Differentiation**

1

(a)

$\bullet^1$ $y_{x=-2} = 10$

$\bullet^2$ $\frac{dy}{dx} = \ldots\ldots$

$\bullet^3$ $3x^2 - 9$

$\bullet^4$ $m_{x=-2} = 3$

$\bullet^5$ $y - 10 = 3(x+2)$

(b)

$\bullet^6$ $y = 3x + 16$

$\bullet^7$ $3x + 16 = x^3 - 9x$

$\bullet^8$ $x^3 - 12x - 16 = 0$

$\bullet^9$ *e.g.*

-2	1	0	-12	-16
		-2	4	16
	1	-2	-8	0

$\bullet^{10}$ *e.g.* $x^2 - 2x - 8$

$\bullet^{11}$ *e.g.* $(x+2)(x-4)$

$\bullet^{12}$ B is $(4, 28)$

2

(a)

$\bullet^1$ $\frac{dy}{dx} = \ldots\ldots\ldots$

$\bullet^2$ $3x^2 - 4x + 1$

$\bullet^3$ $m_{x=2} = 5$

$\bullet^4$ $y - 2 = 5(x-2)$

(b)

$\bullet^5$ equate 'y's

$\bullet^6$ $x^3 - 2x^2 - 4x + 8 = 0$

$\bullet^7$ e.g. synthetic division

$\bullet^8$ the appearance of:

$x^2 - 4$

or $x^2 - 4x + 4$

or ± 2

or $-2, 2, 2$

$\bullet^9$ $x = -2,\ y = -18$

3

(a)

$\bullet^1$ any linear factor

$\bullet^2$ corresponding quadratic factor

$\bullet^3$ $f(x) = (x+1)(x-4)(x+4)$

(b)

$\bullet^4$ For all 3 points on x-axis

$\bullet^5$ $(0, -16)$

(c)

$\bullet^6$ $f'(x) = 3x^2 + 2x - 16$

$\bullet^7$ use $f'(x) = 0$

$\bullet^8$ $x = 2$, **and** $x = -\frac{8}{3}$

$\bullet^9$ $y = -36$, **and** $y = \frac{400}{27}(14.8)$

$\bullet^{10}$

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	+
	⋰	…	⋱	⋱	…	⋰

$\bullet^{11}$ max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2, -36)$